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MATHEMATICS IN AMERICA

American Journal of Mathematics, Pure and Applied. Published under the Auspices of the Johns Hopkins University. Vols. III. and IV. (Baltimore: Isaac Friedenwald.) And other Mathematical Journals.

THE *American Journal of Mathematics* was established in 1878 under the auspices of the Johns Hopkins University at Baltimore, and four handsome quarto volumes of 400 pages each have now been published. Prof. Sylvester was editor-in-chief of the first three volumes, being assisted by Mr. Story as editor-in-charge, but the last volume bears Sylvester's name alone as editor.

A notice of the first two volumes of the Journal appeared in NATURE, vol. xxii. p. 73, and the hope was there expressed that it might have as great a future before it as awaited Crelle's Journal half a century before. A careful examination of the last two volumes shows that the promise of the earlier volumes has been so far maintained, and that the Journal has already acquired a distinctive character of its own. It almost invariably happens that mathematical journals exhibit marked characteristics, and that certain branches of the subject occupy a pre-eminent position. One paper leads to another relating to the same questions, and the original bias of a journal is generally due, both directly and indirectly, to its editor, as authors naturally prefer to send contributions where they are more likely to be understood and appreciated. That this is especially the case with the American Journal is what we should expect, as besides being the principal contributor, the editor is professor in the institution with which it is connected, and many of the papers are by his former pupils and colleagues. Although a very distinct tendency is thus evident in the direction of the large group of subjects (and more particularly Higher Algebra and Higher Arithmetic) with which the name of Prof. Sylvester is associated, it is not to be supposed that the Journal has become narrow in its scope. On the contrary, the whole range of mathematical subjects is very fairly represented, as will appear from the following paragraphs, which contain a list of the papers in vols. iii. and iv., an attempt being made to group them to some extent according to subjects.

The arithmological papers are numerous. Prof. Sylvester gives closer limits for a quantity which occurs in Tchebycheff's well-known investigation of the number of primes inferior to any given prime; he contributes also a note on the trisection and quartisection of the roots of unity, and an instantaneous proof of a theorem of Lagrange's on the divisors of a certain quadratic form. In a paper on a point in the theory of vulgar fractions he gives a method of developing any vulgar fraction as a sum of certain special fractions, each having unity as its numerator. This development he terms a sorites, and he remarks that it was suggested to him by the chapter in Cantor's *Geschichte der Mathematik*, which gives an account of the singular method in use among the ancient Egyptians for working with fractions: it was their curious custom to resolve every fraction into a sum of simple fractions according to a certain traditional method, which however only leads in a few simple cases to a sorites. There are two papers by Mr. O. H. Mitchell, both relating to the theory of congruences: one of them contains a generalisation of Fermat's and Wilson's theorems. There is also a short note by Prof. Newcomb on the relative frequency of the occurrence of the digits as leading figures in logarithmic tables.

The contributions to the higher algebra occupy a very conspicuous place. The important tables of the generating functions and ground forms of binary quantics which have been calculated by Prof. Sylvester and Mr. F. Franklin, with the aid of a grant from the British Association, are continued. Mr. Franklin is also the author of a separate paper, in which he gives a consecutive account of the methods, due to Cayley and Sylvester, of calculating the generating functions for binary quantics and thence determining the number of fundamental invariants and covariants of any order and degree. Prof. Sylvester gives a determination of the impossibility of the binary octavic possessing any ground form of degree 10 and order 4. There is a paper on the 34 concomitants of the ternary cubic by Prof. Cayley, who also gives a specimen of a literal table for binary quantics and certain tables for the binary sextic; and there are some notes on Modern Algebra by M. Faà de Bruno, of Turin. Mr. Mitchell and Mr. T. Muir, of Glasgow, give theorems relating to determinants.

Prof. Wm. Woolsey Johnson is the author of a paper on strophoids. The term strophoid has been applied by French writers to a cubic curve, the symmetrical form of which Dr. Booth discussed under the name of the Logocyclic curve. The author gives to the term a more extended signification, and defines a strophoid as the locus of the intersection of two straight lines which rotate uniformly about two fixed points in a plane. Dr. Booth's curve is included as a particular case of the class of curves which Prof. Johnson terms right strophoids. Prof. Sylvester considers the theory of rational derivation on a cubic, and Mr. Story is thus led to discuss the subject more fully in a separate memoir: the points on the curve which are considered are those whose coordinates can be expressed as rational functions of an arbitrary initial point on the curve. Mr. Samuel Roberts contributes a paper on the generalisation of local theorems, in which the generating point divides a variable linear segment in a constant ratio; and there is a note by Miss Christine Ladd on segments made on lines by curves.

There are three papers on solid geometry, all by Mr. T. Craig: they relate to the orthomorphic projection of an ellipsoid upon a sphere, to certain metrical properties of surfaces (in n dimensions as well as in three dimensions), and to the counter-pedal surface of an ellipsoid. The surface which the author designates the counter-pedal is the locus of the intersections of central planes parallel to the tangent planes of the ellipsoid with the normals at the corresponding points of contact; its equation is worked out, and is found to be of the tenth order. Mr. E. W. Hyde contributes a note on the centre of gravity of a solid of revolution, and there is a discussion by Prof. Stringham on the regular figures in n-dimensional space.

Prof. Cayley gives a note on the analytical forms or

ramifications which he has termed trees. This is a subject which has applications to the theory of chemical combinations, and it is one to which Prof. Cayley has already devoted attention, a long memoir of his upon it having appeared in the Report of the British Association for 1875. Prof. Cayley also is the author of a short paper on certain imaginaries connected with the product of two sums of eight squares. Among papers on general analysis should also be mentioned a determination by Prof. Stringham, of the number of possible finite groups of quaternions, a group being defined as in the theory of substitutions; and a note by Mr. Story on non-Euclidean geometry.

A number of papers relate to the differential calculus. There are two by Prof. Sylvester on the solution of classes of difference and differential equations. No less than four relate to development in series by Taylor's and other expansion theorems and the forms of the remainder: these are by Mr. J. C. Glashan of Ottawa, Mr. McClintock, and Mr. A. W. Whitcom. Prof. Crofton gives some remarkable theorems involving symbols of operation, and Mr. J. Hammond considers the theory of general differentiation, a subject that received attention from Liouville, Peacock, and others half a century ago, but which has attracted but little notice in recent times. Mr. Franklin contributes a short note on Newton's method of approximating to the roots of equations, and Mr. Glashan gives certain formulæ relating to the change of the independent variable in differentiations.

More than one whole number is devoted to a reprint of the late Prof. Benjamin Peirce's valuable memoir on Linear Associative Algebra, of which only a small number of copies in lithograph were made in the author's lifetime for circulation among his friends. This well-known paper was read before the National Academy of Sciences at Washington in 1870: it is here reproduced with notes and addenda by Mr. C. S. Peirce, the son of the author, and occupies 133 pages. Mr. C. S. Peirce himself, who is known not only by his logical writings but by his stellar photometric researches, is also the author of two papers, the one on the algebra of logic and the other on the logic of number. In connection with this subject a paper by Miss Ladd on De Morgan's extension of the algebraical processes should be noticed.

There is a short bibliographical paper relating to Alhazen's problem by Mr. Marcus Baker. The problem is from two points in the plane of a circle to draw lines meeting at a point on the circumference, and making equal angles with the tangent at that point. The author also gives an extension of the problem to the sphere.

Only three papers belong to mathematical physics. One, by Prof. H. A. Rowland, relates to the general equations of electromagnetic action, with application to a new theory of magnetic attractions and to the theory of the magnetic relation of the plane of polarisation of light. The other two are on hydrodynamics, they are by Prof. Rowland and Mr. Craig, and relate respectively to the motion of a perfect incompressible fluid and to certain possible cases of steady motion in a viscous fluid. There are some notes on moving axes by Prof. Loudon of Toronto; and Prof. Sylvester considers the theory of mechanical involution, which is the name he has given to the relation between six lines in space so situated that forces may be made to act along them whose statical sum is zero. Linkages form the subject of a paper by Mr. F. T. Freeland.

Astronomy is represented by two papers: one by Prof. Newcomb, on a method of developing the perturbative function of planetary motion; and the other by Mr. G. W. Hill, on Hansen's general formulæ for perturbations. The object of the former paper is to exhibit a method of effecting the development in powers of the eccentricities. The author remarks that in consequence of the complex character of the series this development has been but little used even in the cases of nearly circular orbits, when its application would be most convenient, but that, as the disturbing force is given as an explicit function of all the elements, it is of more interest to the mathematician than any other. In his method of development Prof. Newcomb directs especial attention to the expression of the coefficient of each power of the eccentricity in terms of the coefficients of lower powers, and to the expression of the coefficient in each term involving the perihelia of two planets as the symbolic product of coefficients involving the perihelion of one only. Mr. Hill's paper contains a transformed form of Hansen's expression for the perturbation of the mean anomaly, which is more simple and more convenient for computation. There is also a short note by Mr. Ormond Stone relating to formulæ in elliptic motion.

The titles of the papers speak for themselves, and but little comment is needed. It will be seen that the two volumes represent a considerable amount of mathematical work, a fair proportion of which may have real influence on the advancement of the science. Some of the papers, as must evidently be the case, are needlessly pretentious in form, and the new matter they contain might be advantageously stated in less space. The effect of Professor Cayley's visit to Baltimore is apparent in the papers which occur in the last number issued, and we believe that the lectures which he gave at the Johns Hopkins University will shortly appear in a future number.

The dates which the numbers of the Journal bear are the dates when they ought to have appeared, assuming it to be published quarterly in March, June, September, and December, and not the dates when they did actually appear. Thus the last number issued bears date December, 1881, but in the case of this number the inconvenience attending so great a discrepancy between the nominal date and the date of publication is partially remedied by the words "Issued July 18, 1882," at the foot of the last page. It seems a pity to retain the nominal dates on the wrappers, as they may be misleading. It will be difficult to regain the lost time, and there is but little advantage in stating the time when the number should have appeared. The volume and number and date of publication are all that need to be given.

On the wrappers of the numbers of vol. iv. appears an announcement in which a prize of 1500 francs and a perpetual free subscription to the journal from its commencement are offered to the first person who, before January 1, 1883, discovers and transmits to the Editor a valid proof or disproof of the proposition that a ground form and a syzygant of the same degree and order cannot appertain

to the same binary quantic, "provided that the Editor shall not himself have previously discovered the same, and given public notice thereof." The truth of this proposition has been assumed as a fundamental postulate in the calculation of ground forms, and its importance cannot be over-estimated. It is, however, somewhat of an anachronism to draw attention to it by the offer of a prize. Such prizes exist in Universities and in the older academies, but by many they are not regarded with much favour. It seems unlikely that any competent person would be tempted to investigate the subject by hope of the reward. Pure mathematics offers no mercenary inducements to its followers, who are attracted to it by the importance and beauty of the truths it contains; and the complete absence of any material advantage to be gained by means of it, adds perhaps even another charm to its study.

The late Prof. Benjamin Peirce denoted the base of the Napierian logarithms and the ratio of the circumference to the diameter of a circle by two special symbols turned opposite ways, somewhat resembling a 6 and a 6 reversed. The forms of these symbols would seem to imply that 2·71828... and 3·14159... were regarded as allied to one another, and in some reciprocal or inverse manner too, though it is not easy to see what the author's point of view was. Two writers in the *American Journal* use Prof. Peirce's symbols in place of π and e, and this is to be regretted, as any departure from the recognised notation in elementary matters is always unfortunate. Even if the symbols were happily chosen, which does not appear to be the case, they would require the cutting of new type, and it is absolutely certain that there is not the least chance of their general adoption. If now used by a few prominent writers in America, they may spread to such an extent as to make it very difficult for their successors to get rid of them. The preservation of the international character of mathematical notation is of paramount importance, and the existence of local notations, especially when they find their way into text-books, is a calamity. In England the Cambridge notations, $\sin^{-1}x$, due we believe to Herschel and Babbage, and the factorial notation due to the late Prof. Jarrett, are still retained by many English writers, although it has long been evident that there is no chance of their adoption by continental mathematicians. It is always desirable to adhere to an established notation, if it is generally understood and accepted, even if it is unsatisfactory, rather than attempt to replace it by a better one, unless there seems very good reason to suppose that the attempt will be successful.

In the previous article in NATURE reference was made to the services which Dr. J. E. Hendricks, of Des Moines, Iowa, has rendered to mathematics in America by the publication of the *Analyst*, which he established in 1874, and has continued to the present time. This journal is published every two months, and has now completed its ninth volume. In spite of typographical and other difficulties the editor has published it regularly, and it shows no signs of diminished vitality or interest on his part. It has been self-supporting, and its success is due to the genuine love of their subject felt by the editor and the contributors. A great part of each number is unfortunately devoted to problems—the lowest form of mathematics—and the space available for more valuable matter is thus considerably diminished. One also is tempted to wish that the editor would show greater strictness in curtailing or excluding the writings of certain contributors, but nevertheless the *Analyst* contains not a few useful papers. It is easy to see the blemishes in such a journal by merely turning over the pages, but it is not so easy to estimate the services which it confers upon science by inducing teachers to look beyond the text-books and interest themselves in a subject for which a genuine taste can only be acquired by attempting to do work for oneself. The large quarto page of the *American Journal* and the elaborate nature of some of its papers render it unsuitable for the short notes and the more unpretentious class of papers in which the author lays but little claim to originality. For these the *Analyst* is available; but, after all, the chief value of such a publication consists in the interest in mathematics it excites and fosters in those who could be reached in no other way, and the inducement it affords for those who are unable to devote their whole time to the subject to nevertheless undertake useful and profitable work. No previous American mathematical journal has ever been published regularly for nine years, and Dr. Hendrick has reason to feel proud of the success of his efforts.

In 1877 Mr. Artemas Martin, of Erie, Pennsylvania, issued the first number of the *Mathematical Visitor*, a quarto journal which was published annually until Jan., 1880, and since then has appeared semi-annually. The first volume ended with the number published in January, 1881. The journal consists entirely of problems and solutions, there being a senior and a junior department. Several of the problems relate to probability questions and involve very complicated and elaborate integrations. As the solution of a prize question, Mr. E. B. Seitz gives, in the number for January, 1879, the values of the coefficients obtained by reverting a general series proceeding by ascending powers of the variable, as far as the sixteenth order. The journal is beautifully printed, and is set up by the editor himself. In the number for January, 1880, he says: "This number of the *Visitor* has been delayed some months, in consequence of the sickness of the editor, who has done all the type-setting with his own hands. He is not a practical printer, and never had set up a stickful of type till last May or June."

At the beginning of the present year Mr. Martin issued, besides the *Visitor*, a new publication, entitled *The Mathematical Magazine: a Journal of Elementary Mathematics*, of which four numbers have now appeared. It is of large quarto size, and, like the *Visitor*, is printed by the editor's own hands. No mathematical journal, if it is to contain anything of real value, can be elementary. Mathematics is an old science, and the really elementary parts of it must be acquired from text-books and by means of the examples which the student works out for himself as exercises. Elementary mathematics is a subject for the school-room, but is unsuitable for a journal, and such a publication as Mr. Martin's, if it continues elementary, is educational rather than scientific. In no instance, we believe, has it been found possible to restrict a mathematical journal really to the elements of the subject alone, though of course elementary articles, and articles which are of interest to junior readers, form

a considerable portion of each number of some mathematical publications. Mathematical investigations that are really valuable can never be made elementary, and the questions that can be treated by elementary mathematics are too trivial to deserve recognition in a scientific journal.

We may notice a demonstration of Euclid i. 47, by the late General Garfield, which appears in the first number. If the figure is completed it is in fact an intuitive geometrical proof that $(a+b)^2 = c^2 + 2ab$, where a and b are the sides and c the hypotenuse of a right-angled triangle. The construction is to divide the four sides of a square each into two parts, a and b, in the same order, and to join the points of division. Each of the joining lines is thus equal to the hypotenuse c, and the whole square, $(a+b)^2$, is evidently equal to the inside square, c^2, and the four triangles in the corners, each of which is equal to $\frac{1}{2}ab$. The figure is practically the same as in the well-known proof in which the squares a^2 and b^2 are placed side by side and divided by only two lines in such a manner that the parts may be moved by mere translation (without rotation) so as to form the square c^2, but the special features which give this proof its remarkable elegance are absent. Garfield's proof is Indian in its character, and must have been known to Bhascara, but in the rather more elegant one given in the Vija Ganita (1150) the lines are drawn from the angles of the square c^2 parallel to the sides of the triangle, and include a square $(a-b)^2$, each of the triangles in the corners being $\frac{1}{2}ab$ as before, so that the theorem proved is $c^2 = (a-b)^2 + 2ab$. If the points of division in the figure in § 150 of the Vija Ganita, in which it is shown that $(a+b)^2 - 4ab = (a-b)^2$ are joined, the figure includes both Garfield's and the Hindoo constructions. The construction given by Garfield must have been of course discovered over and over again, and, on its own account, it is so self-evident as only to be interesting historically in connection with the Indian proofs.

If therefore we include among journals one published at such long intervals as half a year there are now no less than four journals, devoted exclusively to mathematics, published in the United States.

With reference to the list of mathematical journals given in the previous article in NATURE, it may be mentioned that the Belgian journal, the *Nouvelle Correspondance Mathématique*, which was edited by M. Catalan, with the co-operation of MM. Mansion, Brocard, Neuberg, and others, was discontinued at the end of 1880. It has been replaced by a new journal, *Mathesis*, which has since been published monthly under the editorship of MM. Mansion and Neuberg, and has now completed its second volume. In this journal the titles of elementary articles are marked by a cross; there are not on the average more than one or two so marked in each number.

A new Scandinavian mathematical journal is shortly to appear under the editorship of Prof. H. G. Zeuthen, of Copenhagen, and Prof. Mittiag-Leffler, of Stockholm. It is to be hoped that it has a great scientific career before it, and assuredly no journal will bear on its title-page the names of more illustrious mathematicians, or will have started under more favourable auspices.

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QUAIN'S "ANATOMY"

Quain's Elements of Anatomy. Edited by Allen Thomson, E. A. Schäfer, and G. D. Thane. Two volumes. Ninth edition. (London: Longmans, Green and Co., 1882.)

Lehrbuch der Neurologie. Fortsetzung von Hoffmann's "Lehrbuch der Anatomie." Von Dr. G. Schwalbe. (Erlangen: Eduard Besold, 1880 and 1881.)

THE appearance of a new edition of Quain's Anatomy is always regarded with attention and interest by teachers of anatomy. The high reputation of its successive editors, Richard Quain, William Sharpey, G. V. Ellis, Allen Thomson, and John Cleland, and the care which has been taken to revise each edition and to incorporate with it the latest additions to anatomical knowledge, have caused this work to be universally regarded as an authority, and have gained for it the position of a standard treatise on Human Anatomy.

The new edition, the ninth, which has just appeared, has been prepared under the editorial supervision of Professors Schäfer and Thane, and Dr. Allen Thomson. The first volume, which has been revised by Prof. Thane, contains the descriptive anatomy of the bones, joints, muscles, blood-vessels, but not the heart; cerebro-spinal and sympathetic nerves, but not the brain and spinal cord; with a chapter on superficial and topographical anatomy, in which the editor has been assisted by Mr. R. J. Godlee. The second volume has been for the most part revised by Mr. Schäfer, and contains the histology, and the anatomy of the viscera, including the heart and central organs of the nervous system; whilst a special chapter on embryology has been written by Dr. Thomson.

The separation of the anatomy of the heart from that of the other parts of the vascular system, as well as of the anatomy of the brain and spinal cord from the nerves which arise from them, and from the sympathetic system, both of which are so intimately connected both anatomically and physiologically with both brain and cord, was first made in the eighth edition; for prior to that time they had always been described along with, and as parts of their respective systems. This arrangement, which is also carried out in the present edition, is, in our judgment, most unphilosophical, for it both destroys the continuity of description, and leads the student to dissociate in his mind the origin of the nerves and blood-vessels from their distribution. Such a dissociation might indeed, as regards the nervous system, have been excusable at the time when both the distributory portions of the cranial and spinal nerves and the sympathetic system were believed to be developed quite independently of the cerebro-spinal axis, and only to become connected with it secondarily. But now-a-days, since through the researches, more especially of the much-lamented F. M Balfour, both the cranial and spinal nerves and the sympathetic have been shown to be true offshoots of the cerebro-spinal axis, and like it of epiblastic origin, to dissociate them, even for descriptive purposes, in a systematic text-book, is, we believe, injurious to real progress. The editors of "Quain" would, we suppose, scarcely think of describing in one volume the gangliated cord of the sympathetic, and in another the nerves which arise